Comparing F-theory Standard Model Constructions

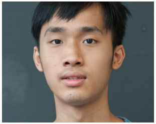
String Pheno 2022
Liverpool, England

Wednesday, July 6, 2022

Washington (Wati) Taylor, MIT

Based in part on recent and upcoming work with:

Patrick Jefferson



Shing Yan (Kobe) Li

Andrew Turner

Outline

1. Possible realizations of the Standard Model in F-theory
2. Which constructions are most typical/natural?
3. Which constructions match observed physics best?

General philosophy of this talk

– Start from the top down.
F-theory gives the broadest global picture known of 4D $\mathcal{N} = 1$ string vacua.

– First: different ways of realizing the SM gauge group, chiral matter

– Second: compare these approaches.

F-theory: Nonperturbative formulation of type IIB string theory

Dictionary for geometry $\leftrightarrow$ physics [Vafa, Morrison-Vafa]

$\sim$ compactification of IIB on compact Kähler (non-CY) space B (e.g. $\mathbb{P}^n$)

B_2 (complex surface) $\to$ 6D, $B_3 \to$ 4D.

Elliptic fibration: $\pi : X(CY) \to B$,
$\pi^{-1}(p) \cong T^2$, for general $p \in B$

Fiber singularities $\to$

Gauge group G (codimension 1 in B)

Matter (codimension 2 in B)

Defined by Weierstrass model (fiber τ = 10D IIB axiodilaton)

$$y^2 = x^3 + fx + g, \quad f, g \text{ “functions” on } B_2$$

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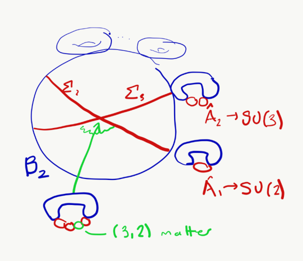

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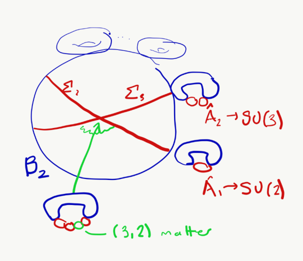

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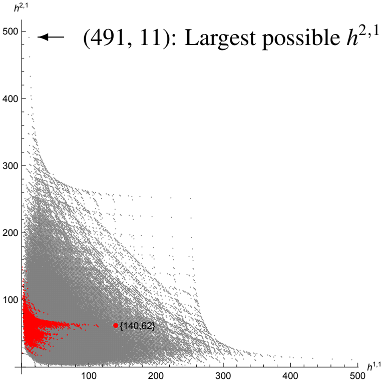
Some preliminary global features of the F-theory landscape

Most known Calabi-Yau threefolds and fourfolds are elliptic
(Empirical results, theoretical arguments: [Huang/WT, Anderson/Gao/Gray/Lee])
KS: all but red ones [$\sim$ 30k/400M] admit elliptic/g1 fibration



Set of elliptic Calabi-Yau threefolds bounded, finite, well-described
Similar for CY4, less complete classification but best global landscape picture

Rigid (non-Higgsable) gauge groups [(Morrison/WT)^2]

In 6D and 4D, most bases force geometrically non-Higgsable G
IIB: 7-branes nucleate on *rigid* loci w/ negative normal bundle

Rigid gauge factors (4D): $SU(2), SU(3), G_2, SO(7), SO(8), F_4, E_6, E_7, E_8$
Note, however, not SU(5)

Products of two factors with joint matter (4D):
$G_2 \times SU(2), SO(7) \times SU(2), SU(2) \times SU(2), SU(3) \times SU(2), SU(3) \times SU(3)$

Rigid/non-Higgsable clusters only interact through gravity, scalars,
provide natural dark matter candidates

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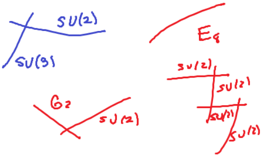

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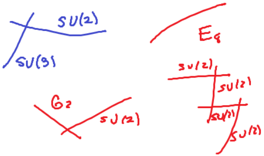

F-theory approaches to the standard model

There are many different ways the standard model may be realized in F-theory

Focus on distinct realizations of SM gauge group from F-theory geometry

	GUT	$SU(3) \times SU(2) \times U(1)$
Tuned G	Tuned GUT (e.g., $SU(5)$)	Direct tuned G_{SM}
Rigid G	Rigid GUT (e.g., E_6, E_7)	Rigid G_{SM}

Construction I: Tuned GUT e.g. SU(5)
[Beasley/Heckman/Vafa, Donagi-Wijnholt, . . .]

- Start with tuned SU(5) in Weierstrass model
- Break with hypercharge flux (requires *remainder* flux in $H^{2,2}_{\text{rem}}(X, \mathbb{Z})$) to get SM chiral spectrum.

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Construction II: Tuned $G_{\text{SM}} = (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$:

- G_{SM} models from toric fiber "F_{11}"
  [Klevers/Mayorga Peña/Oehlmann/Piragua/Reuter,
  global aspects: Grimm/Kapfer/Klevers, Cvetič/Lin]

- + SM chiral matter: e.g. "Quadrillion Standard Models" from $\mathbb{F}_{11}$
  [Cvetič/Halverson/Lin/Liu/Tian]

+ vector matter: [Bies/Cvetič/(Donagi/Liu/Ong)]^4

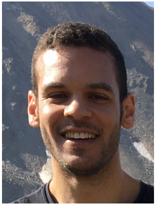
- Universal G_{SM} Weierstrass model ($\mathbb{F}_{11}$ model special case)
  [(Raghuram)/WT/Turner]^2

+ chiral matter [Jefferson/WT/Turner]^{2*}

– Generically gives SM chiral matter + 2 families exotics

[Note: for tuned $SU(3) \times SU(2) \times U(1)$ SM representations are highly non-generic (fine tuning to very exotic singularities)]

Interlude: Middle cohomology on elliptic Calabi-Yau fourfolds and chiral matter in 4D F-theory models



Patrick Jefferson

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Based on arXiv:2108.07810, 2207.nnnnn? by P. Jefferson, WT, A. Turner

Topology of elliptic Calabi-Yau fourfolds

Hodge numbers for elliptic CY fourfold

$h^{1,1}(X) = h^{1,1}(B) + \text{rk } G + 1$ (STW); Divisors D_0, $D_\alpha = \pi^* D_\alpha^{(B)}$, D_i Cartan

$h^{3,1} = \#$ complex structure moduli, $h^{2,1}$ generally 0 or small

$h^{2,2} = 4(h^{1,1} + h^{3,1}) + 44 - 2h^{2,1}$, $\chi = 6(8 + h^{1,1} + h^{3,1} - h^{2,1})$

For fluxes and chiral matter, we are interested in *vertical* cohomology

$$H_{2,2}^{\text{vert}} = \text{span}_{\mathbb{Z}}(H^{1,1}(X,\mathbb{Z}) \wedge H^{1,1}(X,\mathbb{Z}))$$

Denote $S_{IJ} = D_I \cap D_J$; note, homology relations $\to$ linear dependencies

Fluxes in $H_{2,2}^{\text{vert}} \to$ chiral matter

$H^4(X)$ has orthogonal decomposition [Greene/Morrison/Plesser, Braun/Watari]

$$H^4(X,\mathbb{C}) = H^{2,2}_{\text{vert}}(X,\mathbb{C}) \oplus H^{2,2}_{\text{rem}}(X,\mathbb{C}) \oplus H^4_{\text{hor}}(X,\mathbb{C})\,.$$

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Chiral matter in 4D F-theory models

Flux: $G_{\mathbb{Z}} = G - \frac{c_2(X)}{2} \in H^4(X, \mathbb{Z})$ [Witten]

Satisfies various conditions

SUSY $\Rightarrow G \in H^{2,2}(X, \mathbb{R}) \cap H^4(X, \mathbb{Z}/2), J \wedge G = 0$ [Becker[2], GVW]

Tadpole: $N_{\text{M2}} = \frac{\chi}{24} - \frac{1}{2}\int_X G \wedge G \in \mathbb{Z}_{\geq 0}$ [SVW, DM, DRS]

Poincaré invariance: $\int_{S_{0\alpha}} G = 0\,, \quad \int_{S_{\alpha\beta}} G = 0$

Gauge symmetry preserved: $\int_{S_{i\alpha}} G = 0$ (for E_7 breaking will be $\neq 0$!)

Chiral matter is determined by fluxes, primarily through vertical cycles

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[Donagi/Wijnholt, Beasley/Heckman/Vafa, Braun/Collinucci/Valandro, Marsano/Schäfer-Nameki,Krause/Mayrhofer/Weigand, Grimm/Hayashi]

For some G over general surfaces:
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Intersection form on middle cohomology

Previous work on chiral matter in F-theory models used explicit resolutions

Our approach identifies a resolution-independent structure allowing systematic and base-independent analysis for many gauge groups

Basic idea:

M_{IJKL} intersection numbers on CY4 X generally depend on resolution.

Organize as matrix on $H_{2,2}^{\text{vert}}$: $\quad M_{(IJ)(KL)} = M_{IJKL} = S_{IJ} \cdot S_{KL}$.

We then have fluxes $\chi_R \sim \Theta_{IJ} = \int_{S_{IJ}} G = M_{(IJ)(KL)} \phi^{KL}$,
where $G = \sum_{KL} \phi_{KL} \, \text{PD}(S_{IJ})$.

Removing the null space associated with trivial homology elements,

$$M \to M_{\text{red}} \quad \text{is nondegenerate}$$

Observation/conjecture: M_{red} is resolution independent up to basis
(seen in large classes of examples, general argument with one assumption)

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Explicit form of $M_{\rm red}$

Can compute general form of $M_{\rm red}$ for various gauge groups over general bases, using systematic approach to resolution building on earlier work [Esole/Jefferson/Kang]

e.g. simple nonabelian G in basis $S_{0\alpha}, S_{\alpha\beta}, S_{i\alpha}, S_{ij}$

$$M_{\rm red} = \begin{pmatrix} D_{\alpha'} \cdot K \cdot D_\alpha & D_{\alpha'} \cdot D_\alpha \cdot D_\beta & 0 & 0 \\ D_{\alpha'} \cdot D_{\beta'} \cdot D_\alpha & 0 & 0 & * \\ 0 & 0 & -\kappa^{ij} \Sigma \cdot D_\alpha \cdot D_{\alpha'} & * \\ 0 & * & * & * \end{pmatrix} .$$

or after a (non-integral) change of basis

$$U^{\rm t} M_{\rm red} U = \begin{pmatrix} D_{\alpha'} \cdot K \cdot D_\alpha & D_{\alpha'} \cdot D_\alpha \cdot D_\beta & 0 & 0 \\ D_{\alpha'} \cdot D_{\beta'} \cdot D_\alpha & 0 & 0 & 0 \\ 0 & 0 & -\kappa^{ij} \Sigma \cdot D_\alpha \cdot D_{\alpha'} & 0 \\ 0 & 0 & 0 & \frac{M_{\rm phys}}{(\det \kappa)^2} \end{pmatrix} ,$$

where $M_{\rm phys}$ encodes physics of chiral matter + fluxes, $(000\chi) = M_{\rm red} \cdot [G]$.

F-theory Standard Model constructions, continued

Construction III: Non-Higgsable $G_{\rm SM}$

- $\mathrm{SU}(3) \times \mathrm{SU}(2)$ can be geometrically non-Higgsable in 4D
  [Grassi/Halverson/Shaneson/WT]
- Rigid U(1) factor difficult, however, to integrate
  [Martini/WT, Wang]

Construction IV: rigid GUT

- Breaking $E_7, E_6 \to G_{\rm SM}$ with fluxes

– Using base and resolution-independent form of $M_{\rm red} \to$ systematic analysis
  [Li/WT][2*], see SYL talk

– Chiral matter including SM spectrum can arise even from E_7

– Small number of generations, including 3, arise naturally

- $G_{\rm SM} \subset E_8$ suggests natural realization
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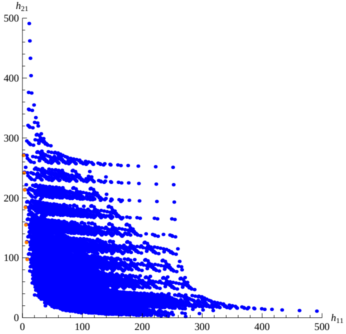
2. Which constructions are most typical/natural?

- A. Prevalence of rigid/non-Higgsable gauge groups

6D SUGRA/F-theory: One large moduli space of connected branches



[All but orange branches contain NHC's, $\sim$ 61000 toric bases]

Typical $G : E_8^5 \times F_4^6 \times (G2 \times SU(2))^{10}$;

4D: similar story; $\sim 4000/10^{3000}$ (weak Fano) bases lack NHC's

- B. Tuning issues

– Tuned models are rare in landscape: require tuning many moduli, many bases will not support [Braun/Watari]

Observations A (ubiquitous rigid G's) + B (difficulty tuning)
$\rightarrow$ suggest that constructions of type I, II are very special/fine-tuned.

Caveat: While exponential factors are overwhelming, many open questions about proper measure in landscape: in particular e.g. how to factor in flux degeneracy (e.g. [WT/Wang]), triangulation degeneracy (e.g. [Demirtas/Long/McAllister/Stillman, Wang])

This suggests that models based on rigid groups are most typical/natural

However, type III models with rigid G_{SM} are also difficult to construct

– Rigid U(1) factors require very special bases
[Martini/WT, Morrison/Park/WT, Wang]

This motivates more work on breaking rigid GUT $\rightarrow G_{\text{SM}}$

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Further issues

From the story so far, flux breaking of a rigid GUT seems overwhelmingly less fine tuned. But there are some further complications:

Basically, while we have some good hints we need a better understanding of the global space of elliptic Calabi-Yau fourfolds

– For elliptic CY threefolds, at large Hodge numbers, toric geometry and the KS hypersurface database give a reasonably representative sample [WT/Wang].

– Monte Carlo sampling of toric threefold bases (including triangulation redundancy) suggests $\sim 10^{3000}$ distinct base geometries (not even including fluxes) [Halverson/Long/Sung, WT/Wang], one base actually has $\sim 10^{45000}$ flop phases [Wang].

– In these ensembles rigid E_8 factors dominate, along with F_4 and $G_2 \times SU(2)$, though E_7, E_6 factors seem to arise in an order one fraction ($\sim 20\%$) of bases.

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Further issues, continued

Another issue is that for the $SU(5)$ tuned GUT models and $E_7 \to SU(5) \to SM$ flux breaking models, must have *remainder* flux, requires non-toric base [Beasley/Heckman/Vafa, Donagi-Wijnholt, Blumenhagen/Grimm/Jurke/Weigand, Marsano/Saulina/Schafer-Nameki, Grimm/Krause/Weigand, . . . ; Li/WT])

Remainder flux:

$$G_4^{\text{rem}} = [D_Y|_{C_{\text{rem}}}] \,,$$

where $D_Y = 2D_1 + 4D_2 + 6D_3 + 3D_7$ generates hypercharge, C_{rem} is a curve on Σ (supporting SU(5)), homologically trivial in base B.

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Further issues, continued

Another issue is that for the $SU(5)$ tuned GUT models and $E_7 \to SU(5) \to SM$ flux breaking models, must have *remainder* flux, requires non-toric base [Beasley/Heckman/Vafa, Donagi-Wijnholt, Blumenhagen/Grimm/Jurke/Weigand, Marsano/Saulina/Schafer-Nameki, Grimm/Krause/Weigand, . . . ; Li/WT])

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Exotic matter

Naively, the most natural constructions require extra tuning to remove exotic matter.

• The universal tuned G_{SM} model only requires tuning 2 discrete parameters to remove exotics; the "F_{11}" fiber is a special case with these discrete parameters tuned so all branes align.

• In the flux broken E_7 models, generic breaking to $SU(3) \times SU(2) \times U(1)$ gives very exotic U(1) charges; need intermediate SU(5) breaking to avoid these exotics.

This is of course an ancient problem in phenomenology, that many GUT constructions give multiple exotics, but F-theory seems to raise the question in a sharp and precise fashion where we have at least some sense of quantification in the landscape.

Other questions

• Given these distinct scenarios for constructing SM group + chiral matter, how do the predictions for more detailed SM features compare: Higgs, Yukawa couplings, etc.

• Can we find analogous tremendous statistical hierarchies in dual approaches (e.g. heterotic, M-theory on G_2)?

• Distributions of rigid groups seem similar between 8 supercharges (6D F-theory) and 4 supercharges (4D F-theory). Does this persist to theories with broken SUSY?

• One interesting set of questions relates to how "typical" fluxes affect geometric constructions – for example, if we have a rigid E_7, how likely is it given tadpole constraints that we get breaking to SM, 3 generations, etc. (cf. SYL talk)

Conclusions

• We have at least 4 qualitatively distinct approaches to realizing the Standard Model gauge group and chiral matter content in F-theory.

• New general approach to understanding resolution-independent intersection form on $H^{\text{vert}}_{2,2}$, key for understanding flux compactifications and chiral matter, studying large ensembles of vacua

• Need a better global picture of the set of threefold bases supporting elliptic CY fourfolds.

• For string theory to be good and predictive framework, would hope that at some point certain features of the SM will naturally arise "for free," once some more basic structure is fixed.

These questions may seem rather ambitious but it seems that the global perspective of F-theory puts us in a situation where we may for the first time be able to make some inroads on the perennial question of what is natural in string theory, and whether some features of the Standard Model are "typical" given other components as priors.

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Thank You!!

[THE FOLLOWING SLIDES ARE ALL EXTRA,

NOT PART OF MAIN TALK]

Example: SU(5) chiral matter (see also [Blumenhagen/Grimm/Jurke/Weigand, Grimm/Krause/Weigand, Marsano/Schafer-Nameki, Grimm/Hayashi])

Can compute from M_{red}

$$\Theta_{33} = \Sigma \cdot K \cdot (6K + 5\Sigma)(\phi^{33} - \phi^{35} - \phi^{44} + \phi^{45})/5\,.$$

Using matter surfaces or cnxn to 3D CS couplings ([Cvetič/Grimm/Klevers])

$$\chi_{\mathbf{5}} = -\Theta_{33} = -\chi_{\mathbf{10}}\,.$$

So we have, where generally m is an integer (exceptions e.g. if $5|K$)

$$\chi_{\mathbf{5}} = \Sigma \cdot K \cdot (6K + 5\Sigma)m\,.$$

Base-independent formula for chiral multiplicities
($\sim$ [Cvetič/Grassi/Klevers/Piragua] w/ U(1) factors)

For example for $B = \mathbb{P}^3, \Sigma = nH, -K = 4H$,

$$\chi_{\mathbf{5}} = 4(5n - 24)m$$

Some interesting questions regarding quantization remain

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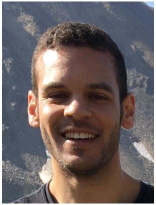
2. Universal tuned standard model structure in F-theory



Patrick Jefferson

Nikhil Raghuram

Andrew Turner

Based on:

arXiv:1906.11092 by WT, A. Turner

arXiv:1912.10991 by N. Raghuram, WT, A. Turner

arXiv:2201.nnnnn? by P. Jefferson, WT, A. Turner

Generic matter for fixed group G: [WT/Turner]

• Matter in highest dimensional branch of (geometric) moduli space; same in 6D, 4D (least tuning)

• Matches simplest singularities in F-theory

• e.g. $SU(N)$: $\{\square, \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}, \text{adjoint}\}$

$SU(3) \times SU(2) \times U(1)$: Standard Model matter not generic (e.g. no $(3, 2)_{q \neq 0}$)

$G_{\text{SM}} = (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$: SM matter + several exotics generic

For given G, generic matter typical, anything else fine-tuned
[Note: more possibilities particularly for U(1) charges when geometric G broken by Higgsing, fluxes]

e.g. $SU(N)$ $\begin{array}{|c|c|}\hline & \\ \hline \end{array}$, SU(2) $\begin{array}{|c|c|c|}\hline & & \\ \hline \end{array}$ possible "exotic" matter in F-theory
[Klevers/Morrison/Raghuram/WT]

Universal G models

For fixed G, matter representations, a *universal G model* is a class of Weierstrass models of full dimensionality (fixed by anomalies in 6D) that geometrically realize G

- Tate models for simple $G = SU(N), E_8, E_7, E_6, F_4, SO(N), G_2, \ldots$
- Morrison-Park model for U(1) with $q = 1, 2$

Universal Weierstrass model for G_{SM} [Raghuram/WT/Turner]

$$
\begin{aligned}
f = &-\frac{1}{48}\left[s_6^2 - 4b_1(d_0 s_5 + d_1 s_2)\right]^2 \\
&+ \frac{1}{2} b_1 d_0 \left[2b_1\left(d_0 s_1 s_8 + d_1 s_2 s_5 + d_2 s_2^2\right) - s_6(s_2 s_8 + b_1 d_1 s_1)\right], \\
g = &\ \frac{1}{864}\left[s_6^2 - 4b_1(d_0 s_5 + d_1 s_2)\right]^3 + \frac{1}{4} b_1^2 d_0^2 \left(s_2 s_8 - b_1 d_1 s_1\right)^2 - b_1^3 d_0^2 d_2 \left(s_2^2 s_5 - s_2 s_1 s_6 + b_1 d_0 s_1^2\right) \\
&- \frac{1}{24} b_1 d_0 \left[s_6^2 - 4b_1(d_0 s_5 + d_1 s_2)\right]\left[2b_1\left(d_0 s_1 s_8 + d_1 s_2 s_5 + d_2 s_2^2\right) - s_6(s_2 s_8 + b_1 d_1 s_1)\right].
\end{aligned}
$$

(Derived from "unHiggsing" Raghuram's U(1) $q = 1, 2, 3, 4$ model)

- Includes "F_{11}" G_{SM} models as a special case

[Klevers/Mayorga Peña/Oehlmann/Piragua/Reuter]

Generic matter for $G_{\rm SM}$ models

	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	$(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$(\mathbf{1},\mathbf{1})_{1}$	$(\mathbf{3},\mathbf{1})_{-\frac{4}{3}}$	$(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$	$(\mathbf{1},\mathbf{1})_{2}$
(MSSM)	1	-1	-1	-1	1	0	0	0
(exotic 1)	2	-1	-4	-2	0	1	0	1
(exotic 2)	-2	2	2	-1	0	0	1	-1

Analysis: [Jefferson/WT/Turner, to appear]

• Generically get all three families from universal model – no constraints from geometry beyond anomaly cancellation

• Closed form formulae for chiral multiplicities χ_i
e.g. $B = \mathbb{P}^3, \Sigma_2 = n_2 H, \Sigma_3 = n_3 H (Y = H)$

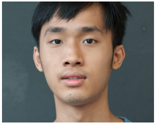
$$\chi_{(3,2,1/6)} = \Theta_{34} = -(11 - n_2 - n_3)(14 - n_2 - 2n_3)\phi_{12}\,.$$

e.g. $n_2 = n_3 : 4|\phi_{12} \to \chi = 396n, n \in \mathbb{Z}$.

• Tuning two discrete parameters gives SM families

• Special case: F_{11} model, recent analysis of 10^{15} 3-generation solutions [Cvetič/Halverson/Lin/(Liu/Tian, Long)]

3. Standard model from E_7 breaking in F-theory



Shing Yan (Kobe) Li

Based on:

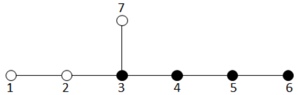
arXiv:2112.03947, 22mm.nnnnn by S.Y. Li, WT

Breaking $E_7 \to G_{\mathrm{SM}}$

Recall

$$\Theta_{IJ} = \int_{S_{IJ}} G = M_{(IJ)(KL)} \phi^{KL} \,.$$

When $\Theta_{i\alpha} \neq 0$, breaks Cartan generator i; $\sum_i c_i \Theta_{i\alpha} = 0 \forall \alpha$ preserves U(1), etc.



Can choose fluxes to break $i = 3, 4, 5, 6$ for any geometric E_7, leaving $SU(3) \times SU(2)$

Note: this realization of $SU(3) \times SU(2)$ is unique up to E_7 automorphism

Depending on fluxes, preserve different $U(1)$ factors, different spectra

– Many $SU(3) \times SU(2) \times U(1)$ breakings, but most have exotics

Intermediate SU(5) and remainder hypercharge flux breaking

To avoid exotics, any appropriate $U(1) \to SU(5)$ enhancement!
(flux vanishes on an additional $\mathbb{P}^1$; equivalent to $\Theta_{3\alpha} = 0$)

Proceed in two steps: 1) Vertical flux breaking $E_7 \to SU(5)$,
2) Remainder flux breaking $SU(5) \to G_{\text{SM}}$
($\sim$ [Beasley/Heckman/Vafa, Donagi-Wijnholt, Blumenhagen/Grimm/Jurke/Weigand, Marsano/Saulina/Schafer-Nameki, Grimm/Krause/Weigand, ...])

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where $D_Y = 2D_1 + 4D_2 + 6D_3 + 3D_7$ generates hypercharge.

C_{rem} is a curve on Σ, homologically trivial in B. Such curves exist on typical non-toric bases [Braun/Collinucci/Valandro]

Matter content with this breaking contains only SM family

$$(\mathbf{3},\mathbf{2})_{1/6}, \quad (\mathbf{3},\mathbf{1})_{2/3}, \quad (\mathbf{3},\mathbf{1})_{-1/3}, \quad (\mathbf{1},\mathbf{2})_{1/2}, \quad (\mathbf{1},\mathbf{1})_{1},$$

arising from (non-chiral) E_7 representations **56** and **133**.

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A simple example (chiral multiplicity for SU(5) only)

We consider the base B a $\mathbb{P}^1$ bundle over Hirzebruch $\mathbb{F}_1$,
Σ an $\mathbb{F}_1$ section with normal bundle $N_\Sigma = -8S - 7F$
(S, F generate divisors of B with $S \cdot S = -1, S \cdot F = 1, F \cdot F = 0$)

$\Rightarrow$ rigid E_7 factor on Σ

To solve the flux constraints in the Kähler cone we need:

$0 > \phi_{iS}/\phi_{iF} = n_S/n_F \neq \infty$ identical for all i

We then have:

$\chi_{(\mathbf{3},\mathbf{2})_{1/6}} = 7n_S + 4n_F, \quad (\phi_{1S}, \phi_{2S}, \phi_{3S}, \phi_{4S}, \phi_{7S}) = (2, 4, 6, 5, 3)n_S \ (+S \to F)$

From $\chi(X) = 51096, h^{2,2}(X) = 34076 \gg \chi(X)/24$, a random flux typically has most entries 0 and small nonzero values.

Minimal solution:

$n_S = -n_F = \pm 1 \Rightarrow$ Number of generations is ± 3

While this is just one example, others have other values, this local structure is ubiquitous in the landscape. Expect similar for geometries with rem flux.

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Full example, using C_{rem} curve

Base B, Σ non-toric. Construction $\sim$ [Braun/Collinucci/Valandro]

Hirzebruch $\mathbb{F}_1 = \mathbb{P}^1$ bundle over $\mathbb{P}^1$, "twist" 1

Define $A = \mathbb{P}^1$ bundle over $\mathbb{F}_1$, "twist" $s + f$ (toric 3-fold)

Define $X = \mathbb{P}^1$ bundle over A, "twist" $a\sigma + bS + cF$, w/ $(a, b, c) = (1, 1, 5)$

Now take B hypersurface with $[B] = \sigma_A + (a+1)\tilde{\sigma} + (b+2)\tilde{S} + (c+3)\tilde{F}$.

Define $\Sigma = B \cdot \sigma_A$.

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- Only 3 independent curves in $A \subset B$, so $\exists C_{\text{rem}}$ for hypercharge flux.

Flux quantization: $\chi_{(\mathbf{3},\mathbf{2})_{1/6}} = -4n_\sigma + n_S + 10n_F$ (n_i signs not all $=$)

$\chi = 3$ from $(1, -3, 1)$, though e.g. $(1, -1, 0) \to \chi = 5$.

Note: analysis purely local, contained in many bases

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Features of $E_7 \to G_{\mathrm{SM}}$ flux construction

- Ubiquitous/natural: construction is possible on typical bases
  estimate 18% of base threefolds have rigid E_7 [WT/Wang]
- Flux breaking of GUT E_7 without its own chiral matter
- No chiral exotics for certain breaking pattern with intermediate SU(5)
- Chiral multiplicity is naturally small.

- Similar construction possible for E_6, more complicated
- Does not work for E_8, but maybe from SCFT matter? [Tian/Wang]

More in upcoming longer paper …

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